Name: Netty THe Incredible

Instructor: ____________________

Math 10170, Exam 2
April 27, 2015

- The Honor Code is in effect for this examination. All work is to be your own.
- You may use your Calculator.
- The exam lasts for 50 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	
8.	
9.	
10.	
11.	
Total	

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Multiple Choice

1.(6 pts.) Let X denote the number of runs in a sequence of five shots by a basketball player with an 80% chance of making a basket on every shot. The probability distribution of X is shown below. What is the expected value of X?

X	P(X)	$xP(x)$
1	0.328	0.328
2	0.2176	0.4352
3	0.3264	0.9792
4	0.1024	0.4096
5	0.0256	0.128

$E(x) = \text{Sum} = 2.28$

~~(a)~~ $E(X) = 2.28$ (b) $E(X) = 4.51$ (c) $E(X) = 3.15$

(d) $E(X) = 1.82$ (e) $E(X) = 1.5$

2.(6 pts.) For a particular martial arts group a match in competition is allowed to last at most five rounds. However, the match may be ended and a winner declared in any round prior to the fifth because of a knockout, a submission or if one competitor is deemed unfit to carry on. The distribution shown below shows the probability distribution of the random variable Y, where Y denotes the number of the round in which a match will end. The expected value of Y is $E(Y) = 3$ What is the standard deviation of Y.

$\mu = E(Y) = 3$

Y	P(Y)	$Y-\mu$	$(Y-\mu)^2$	$(Y-\mu)^2 P(Y)$
1	0.1	$1-3=-2$	$(-2)^2=4$	$4\cdot(0.1)=0.4$
2	0.2	$2-3=-1$	1	0.2
3	0.4	$3-3=0$	0	0
4	0.2	$4-3=1$	1	0.2
5	0.1	$5-3=2$	4	0.4
				1.2

$1.2 = \sigma^2(Y) = \text{Sum}$

(a) $\sigma(Y) = \sqrt{4.2}$ (b) $\sigma(Y) = 4.2$ (c) $\sigma(Y) = 1.2$

~~(d)~~ $\sigma(Y) = \sqrt{1.2}$ (e) $\sigma(Y) = \sqrt{2.2}$

$\sigma(Y) = \sqrt{\sigma^2(Y)} = \sqrt{1.2}$

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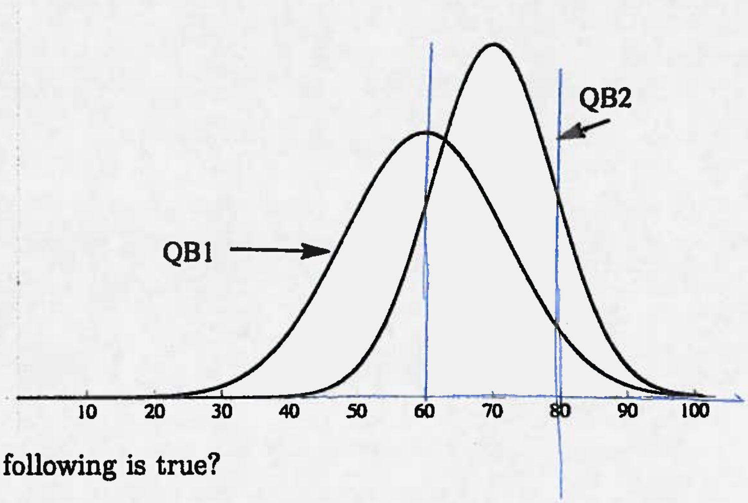
3.(6 pts.) The following picture shows the probability density functions for the quarterback rating per week(a composite statistic) for two NFL Quarterbacks, QB1 and QB2.



Which of the following is true?

(a) The probability that QB2 will have a quarterback rating > 60 on any given week is approximately equal to the probability that QB1 will have a quarterback rating > 60 on that week. AREA UNDE QB2 CURVE to Right of 60 IS > AREA UNDER QB1 CURVE to Right of 60 FALSE

(b) The probability that QB2 will have a quarterback rating > 80 on any given week is approximately 0.6

NO The AREA UNDER QB2 CURVE to Right of 80 is < .5

(c) The probability that QB2 will have a quarterback rating < 60 on any given week is approximately 0.45

NO AREA UNDER QB2 CURVE to LEFT of 60 IS WAY LESS Than .45

(d) The probability that QB1 will have a quarterback rating > 80 on any given week is greater than 0.3

No! THE AREA UNDER QB1 Curve to Right of 80 is WAY Less than 30% of TOTAL AREA UNDER THAT CURVE.

(e) The probability that QB1 will have a quarterback rating < 60 on any given week is approximately 0.5

Yes The AREA TO LEFT of 60 for The QB1 Curve is $\frac{1}{2}$ of the area under the curve (which is 1) So Area to Left of 60 = .5

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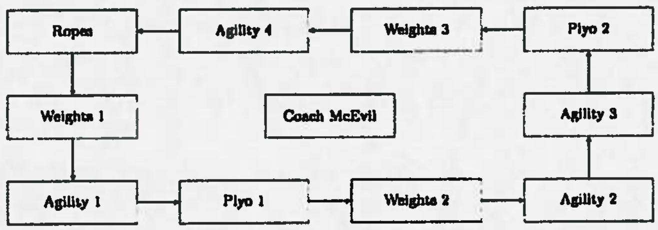
4.(6 pts.) On Circuit training day, Coach McEvil randomly assigns each of his ten players to a different starting point on the circuit.



Three of the circuit training spots have weights, two have plyometric equipment, 4 have agility training equipment and the other spot has climbing ropes. Alfred Agassi is first in line to be assigned to a spot and Bertie Bean is second. What is the probability that both are assigned to start at a spot with weights? (A tree diagram may help you solve this problem.)

(a) 6/90 (b) 9/100 (c) 9/90 (d) 6/10 (e) 1/100

STEP 1
Coach McE
Assigns A.A

STEP 2
Coach McE
Assigns BB
(only 9 spots left)

$\frac{3}{10}$ Weights
- $\frac{2}{9}$ Weights
- $\frac{7}{9}$ Something Else

$\frac{7}{10}$ Something Else
- $\frac{3}{9}$ Weights
- $\frac{6}{9}$ Something Else

P(Both are assigned to Weights)

= Prob. that the squiggly path is followed

$= \frac{3}{10} \cdot \frac{2}{9} = \boxed{\frac{6}{90}}$

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V_i

5.(6 pts.) A ball is thrown directly upwards at a speed of 10 ~~meters per second~~. What is the maximum height above the starting point reached by the ball? (Recall the force of gravity is given by -9.8 meters per second.) ← CAN MAKE $X_i = 0$

STRAIGHT LINE MOTION

2 EQUATIONS

$$\begin{cases} V(t) = V_i - 9.8t \\ \quad = 10 - 9.8t \\ x(t) = x_i + v_i \cdot t - \frac{9.8t^2}{2} \\ \quad = 0 + 10t - 4.9t^2 \end{cases}$$

(a) approx. 9.25 meters

(b) approx. 5.1 meters

(c) approx. 17.2 meters

(d) approx. 1.02 meters

(e) approx. 14.96 meters

@ MAX Height velocity $= 0$

so $10 - 9.8t = 0$; $t = \frac{10}{9.8} = 1.02$ sec.

WHEN $t = 1.02$ sec. Height = MAX Height =

$10(1.02) - 4.9(1.02)^2 = 10.2 - 5.1 = 5.1$ m.

6.(6 pts.) Find equilibrium points of the following payoff matrix for players R and C:

	C1	C2	C3	C4	MAXC
R1	(2,4)	(-1, 3)	(1, 7)	(3, 6)	7
R2	(-2,10)	(6, 7)	(7, 10)	(2, -1)	10
R3	(-2,7)	(4, 8)	(-1, 7)	(5, 10)	10
R4	(3,6)	(7, 9)	(1, 3)	(-1, 2)	9
MAXR	3	7	7	5	

(a) There is a unique equilibrium point at $R1C2$

(b) There is a unique equilibrium point at $R2C1$

(c) There is an equilibrium point at the points $R4C2$ and $R3C4$ only

(d) There are equilibrium points at the points $R2C3$, $R4C2$ and at $R3C4$. only ✓ ✓ ✓

(e) There are no equilibrium points

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7.(6 pts.) In a simplified model of a tennis serve, the server must decide whether to serve to the receiver's forehand (F) or to the backhand (B). The receiver must anticipate whether the serve will come to the forehand (F) or the backhand (B). For players *Robert*(R) and *Carl*(C), it is estimated that

- if Robert serves to the forehand (F) and Carl anticipates correctly, then there is a 50% chance that Robert will win the serve. FF → 50
- On the other hand if Carl does not correctly anticipate the serve to the forehand, there is a 70% chance that Robert will win the serve. FB → 70
- if Robert serves to the backhand (B) and Carl anticipates correctly, then there is a 40% chance that Robert will win the serve. BB → 40
- On the other hand if Carl does not correctly anticipate the serve to the backhand, there is a 60% chance that Robert will win the serve. BF → 60

Which of the following shows the correct payoff matrix for Robert for this constant sum game?

(a)

		Carl F	Carl B
Robert	F	50	40
	B	70	60

(b)

		Carl F	Carl B
Robert	F	40	60
	B	50	70

(c)

		Carl F	Carl B
Robert	F	50	70
	B	60	40

(d)

		Carl F	Carl B
Robert	F	50	30
	B	60	40

(e)

		Carl F	Carl B
Robert	F	70	50
	B	60	40

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Partial Credit

You must show your work on the partial credit problems to receive credit!

8.(12 pts.) The following shows data for 50 consecutive passes for quarterback Drew Brees show whether each pass was complete(C) or incomplete (I):

C C C C | I | C C C | I | C | I I | C | I | C C C C C C C C C C | I
I I | C C | I | C C C | I | C | I I | C C C | I | C C C C C C C C C |

(a) How many runs (of C's and I's) are there in the data?

19 Runs

(b) If X denotes the number of runs in a randomly generated sequence of C's and I's of length N with N_C C's and N_I I's, X has an approximately normal distribution with

$$E(X) = \frac{2N_C N_I}{N} + 1, \quad \text{and} \quad \sigma(X) = \sqrt{\frac{(\mu-1)(\mu-2)}{N-1}}.$$

Applying this distribution to the case given above, what is the Z score of the above observed set of data?

$N = 50$

$N_C = \# C's = 37$

$N_I = \# I's = 13$

$E(x) = \frac{2(37)(13)}{50} + 1 = 20.24 = \mu$

$\sigma(x) = \sqrt{\frac{(19.24)(18.24)}{49}} \cong 2.67.$

Z-Score of our observation of $X \to 19$

$= \frac{19-\mu}{\sigma} = \frac{19-20.24}{2.67} = -0.4644$

(c) Using your knowledge of the empirical rule, would you say that the above data was randomly generated or not? Justify your answer.

If our observation of X was more than 2 standard deviations away from E(x), we would question whether the data was randomly generated (with the same probability of C throughout). However no red flags are raised here since the observed value of X (19) is within one standard deviation of the mean (|z-score| < 1).

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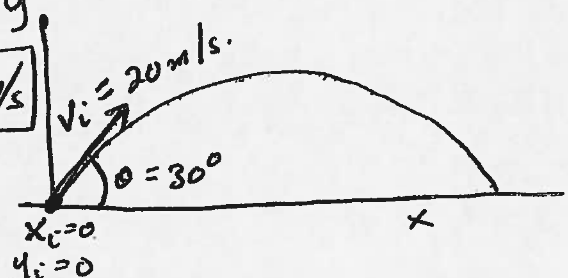
9.(12 pts.) Suppose a soccer player kicks a ball on a level playing field with an initial speed of 20 meters per second at an angle of 30^0 to the horizontal. We assume that the force of gravity ($9.8m/s^2$) is the only force acting on the ball.

(a) Give a formula for the horizontal velocity of the ball after t seconds; ($v_{x,}(t)$), and a formula for the vertical velocity of the ball after t seconds; ($v_y(t)$).

$V_x(t) = V_{ix} = V_i \cos(\theta)$

$= 20 \cos(30^\circ) = 20(.866) = \boxed{17.32\ m/s}$



$V_y(t) = V_{iy} - 9.8\cancel{m/s}\ t$

$= 20 \sin(30^\circ) - 9.8t =$

$20(.5) - 9.8t = \boxed{10 - 9.8t}\ m/s.$

(b) At what time t does the ball reach its maximum height?

Reaches max height when $V_y(t) = 0$

i.e. when $10 - 9.8t = 0$ i.e. when $t = \frac{10}{9.8} = 1.02$ sec.

when $\boxed{t = 1.02 \text{ sec.} = t_{1/2}}$

Not required $\left\{ \begin{array}{l} \text{height} = y(t) = y_i + V_{iy}t - \frac{9.8}{2}t^2 = 10t - 4.9t^2 \\ = 10(1.02) - 4.9(1.02)^2 = 5.1 \text{ meters} \end{array} \right.$

(c) What is the horizontal distance travelled by the ball at the time when it first hits the ground?

Hits ground for first time when $t = 2t_{1/2}$

i.e. $t = 2(1.02) = 2.04$ sec.

The horizontal distance covered @ time t is given by $x(t) = x_i + V_{ix}t$

$= 0 + 17.32t$

Therefore the range is $x(2.04) = 17.32(2.04)$

$= \boxed{35.33 \text{ meters}}$

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10.(14 pts.) Roger and Connor are fencers who frequently face each other in the sabre. The payoff matrix for each bout is shown below, where both players can either attack directly off the line (A) or hold back (H). The payoff for Roger is shown as the expected number of points he will win in the bout for each situation. This is a two-person zero sum game.

		Connor		
	MAX	.5	.8	
		A	H	min
Roger	A	.3 a	.8 b	.3
	H	.5 c	.2 d	.2

Show your work to get credit for the following questions:

(a) Does this matrix have a saddle point?

NO

(b) What is the optimal mixed strategy for Roger?

$(p, 1-p)$ where $p = \dfrac{d-c}{(a+d)-(b+c)} = \dfrac{.2-.5}{(.3+.2)-(.5+.8)} = \dfrac{-.3}{-.8} = \dfrac{3}{8}$

$= \left(\frac{3}{8}, \frac{5}{8}\right)$

(c) What is the optimal mixed strategy for Connor?

$\begin{pmatrix} q \\ 1-q \end{pmatrix}$ where $q = \dfrac{d-b}{(a+d)-(b+c)} = \dfrac{.2-.8}{-.8} = \dfrac{-.6}{-.8} = \dfrac{6}{8}$

$= \cancel{\begin{pmatrix} \frac{6}{8} \\ \frac{4}{8} \end{pmatrix}} \begin{pmatrix} \frac{6}{8} \\ \frac{2}{8} \end{pmatrix}$

(d) What is the value of the game?

$v = \dfrac{ad-bc}{-.8} = \dfrac{(.3)(.2)-(.5)(.8)}{-.8} = \dfrac{.06-.4}{-.8} = \dfrac{-.34}{-.8} = \dfrac{34}{80} \approx \boxed{.425}$

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11.(20 pts.) These 20 points are for the take home part of your exam. You may use this page for rough work.